

# Security Analysis of the Unrestricted Identity-Based Aggregate Signature Scheme

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## Abstract

Aggregate signatures allow anyone to combine different signatures signed by different signers on different messages into a single short signature. An ideal aggregate signature scheme is an identity-based aggregate signature (IBAS) scheme that supports full aggregation since it can reduce the total transmitted data by using an identity string as a public key and anyone can freely aggregate different signatures. Constructing a secure IBAS scheme that supports full aggregation in bilinear maps is an important open problem. Recently, Yuan *et al.* proposed an IBAS scheme with full aggregation in bilinear maps and claimed its security in the random oracle model under the computational Diffie-Hellman assumption. In this paper, we show that there exists an efficient forgery attacker on their IBAS scheme and their security proof has a serious flaw.

**Keywords:** Identity-based signature, Aggregate signature, Security analysis, Bilinear map.

## 1 Introduction

Aggregate signature schemes allow anyone to combine  $n$  different signatures on different  $n$  messages signed by different  $n$  signers into a single short aggregate signature. The main advantage of aggregate signature schemes is to reduce the communication and storage overhead of signatures by compressing these signatures into a single signature. The application of aggregate signature schemes includes secure routing protocols, public-key infrastructure systems, and sensor networks. Boneh *et al.* [5] proposed the first full aggregate signature scheme in which anyone can combine different signatures in bilinear groups and proved its security in the random oracle model. After that, Lysyanskaya *et al.* [14] constructed a sequential aggregate signature scheme such that a signature can be combined in sequential order, and Gentry and Ramzan [7] proposed a synchronized aggregate signature scheme such that all signers should share synchronized information. There are many other aggregate signature schemes with different properties [1, 4, 8, 9, 11–13, 16].

Although aggregate signature schemes can reduce the size of signatures by aggregation, they usually cannot reduce the total amount of transmitted data significantly since a verifier should retrieve all public keys of the signers. Therefore, reducing the size of public keys is also an important issue in aggregate signature schemes [11, 12, 16]. An ideal solution for this problem is to use an identity-based aggregate signature (IBAS) scheme since it uses an already known identity string as the public key of a user [7]. However, there is only one IBAS scheme with full aggregation that was proposed by Hohenberger *et al.* [9] in multilinear maps. The multilinear map is an attractive tool for cryptographic constructions, but it is

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currently impractical since it's basis is a leveled homomorphic encryption scheme [6]. There are some IBAS schemes in bilinear maps, but these IBAS schemes only support sequential aggregation or synchronized aggregation [4, 7, 8]. Therefore, construction an IBAS scheme with full aggregation in bilinear maps is an important open problem.

The main reason for the difficulty of devising an IBAS scheme with full aggregation is that it seems not easy to find a way to aggregate the randomness of all signers in which each randomness of a signer is used to hide the private key of each signer in a signing process [7]. For this reason, current IBAS schemes only support synchronized aggregation or sequential aggregation to aggregate the randomness of all signers [4, 7]. Additionally, designing a secure IBAS scheme is not a easy task since even the original version of Boldyreva *et al.*'s IBAS scheme [3] was broken by Hwang *et al.* [10] and then it was corrected later. Recently, Yuan *et al.* proposed an IBAS scheme with full aggregation in bilinear maps and claimed it security in random oracle models [17]. The authors first proposed an IBS scheme in bilinear maps and constructed an IBAS scheme from the IBS scheme. To prove the security their IBS scheme, the authors claimed that the security of their IBS scheme can be proven under the computational Diffie-Hellman (CDH) assumption by using Forking Lemma in the random oracle model.

In this paper, we show that the IBS and IBAS schemes of Yuan *et al.* are not secure at all. First, we show that there exists a universal forgery attack against the IBS scheme of Yuan *et al.* by using two signatures. This forgery attack also applies to their IBAS scheme. One may wonder that our forgery attack contradicts their claims of the security of the schemes. To solve this, we next show that the security proof of Yuan *et al.*'s IBS scheme has a serious flaw. The security proof of the IBS scheme essentially use the fact that two signatures that are obtained by using Forking Lemma have the same randomness in signatures. However, we show that the forged signatures of an adversary cannot satisfy this condition since the signature of Yuan *et al.*'s IBS scheme is publicly re-randomizable.

This paper is organized as follows: In Section 2, we review bilinear groups and the IBS and IBAS schemes of Yuan *et al.* In Section 3, we present a universal forgery against the IBS scheme. In Section 4, we analyze the security proof of Yuan *et al.*'s IBS scheme and show that the proof has a serious flaw.

## 2 The IBAS Scheme of Yuan *et al.*

In this section, we review bilinear groups and the IBS and IBAS schemes of Yuan *et al.* [17].

### 2.1 Bilinear Groups and Complexity Assumptions

Let  $\mathbb{G}$  and  $\mathbb{G}_T$  be two multiplicative cyclic groups of same prime order  $p$  and  $g$  be a generator of  $\mathbb{G}$ . The bilinear map  $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$  has the following properties:

1. Bilinearity:  $\forall u, v \in \mathbb{G}$  and  $\forall a, b \in \mathbb{Z}_p$ ,  $e(u^a, v^b) = e(u, v)^{ab}$ .
2. Non-degeneracy:  $\exists g$  such that  $e(g, g)$  has order  $p$ , that is,  $e(g, g)$  is a generator of  $\mathbb{G}_T$ .

We say that  $\mathbb{G}$  is a bilinear group if the group operations in  $\mathbb{G}$  and  $\mathbb{G}_T$  as well as the bilinear map  $e$  are all efficiently computable. Furthermore, we assume that the description of  $\mathbb{G}$  and  $\mathbb{G}_T$  includes generators of  $\mathbb{G}$  and  $\mathbb{G}_T$  respectively.

**Assumption 2.1** (Computational Diffie-Hellman, CDH). *Let  $(p, \mathbb{G}, \mathbb{G}_T, e)$  be a description of the bilinear group of prime order  $p$ . Let  $g$  be generators of subgroups  $\mathbb{G}$ . The CDH assumption is that if the challenge tuple  $D = ((p, \mathbb{G}, \mathbb{G}_T, e), g, g^a, g^b)$  is given, no PPT algorithm  $\mathcal{A}$  can output  $g^{ab} \in \mathbb{G}$  with more than a*

negligible advantage. The advantage of  $\mathcal{A}$  is defined as  $\text{Adv}_{\mathcal{A}}^{\text{CDH}}(\lambda) = \Pr[\mathcal{A}(D) = g^{ab}]$  where the probability is taken over random choices of  $a, b \in \mathbb{Z}_p$ .

## 2.2 The Identity-Based Signature Scheme

The IBS scheme consists of **Setup**, **GenKey**, **Sign**, and **Verify** algorithms. The IBS scheme of Yuan *et al.* [17] is described as follows:

**Setup**( $1^\lambda$ ): This algorithm takes as input a security parameter  $1^\lambda$ . It generates bilinear groups  $\mathbb{G}, \mathbb{G}_T$  of prime order  $p$ . Let  $g$  be a random generator of  $\mathbb{G}$ . It chooses random exponents  $s_1, s_2 \in \mathbb{Z}_p^*$  and two cryptographic hash functions  $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}$  and  $H_2 : \{0, 1\}^* \rightarrow \mathbb{Z}_p^*$ . It outputs a master key  $MK = (s_1, s_2)$  and public parameters  $PP = ((p, \mathbb{G}, \mathbb{G}_T, e), g, g_1 = g^{s_1}, g_2 = g^{s_2}, H_1, H_2)$ .

**GenKey**( $ID, MK, PP$ ): This algorithm takes as input an identity  $ID \in \{0, 1\}^*$ , the master key  $MK = (s_1, s_2)$ , and the public parameters  $PP$ . It outputs a private key  $SK_{ID} = (D_1 = H_1(ID)^{s_1}, D_2 = H_1(ID)^{s_2})$ .

**Sign**( $M, SK_{ID}, PP$ ): This algorithm takes as input a message  $M \in \{0, 1\}^*$ , a private key  $SK_{ID} = (D_1, D_2)$ , and the public parameters  $PP$ . It selects a random exponent  $r \in \mathbb{Z}_p^*$  and computes  $h = H_2(ID \| M)$ . It outputs a signature  $\sigma = (U = g^r, V = D_1^h \cdot g_1^r, W = D_2 \cdot g_2^r)$ .

**Verify**( $\sigma, ID, M, PP$ ): This algorithm takes as input a signature  $\sigma = (U, V, W)$ , an identity  $ID \in \{0, 1\}^*$ , a message  $M \in \{0, 1\}^*$ , and the public parameters  $PP$ . It computes  $h = H_2(ID \| M)$  and checks whether  $e(V, g) \stackrel{?}{=} e(H_1(ID)^h \cdot U, g_1)$  and  $e(W, g) \stackrel{?}{=} e(H_1(ID) \cdot U, g_2)$ . If both equations hold, then it outputs 1. Otherwise, it outputs 0.

**Claim 2.2** ([17]). *The above IBS scheme is existentially unforgeable under chosen message attacks in the random oracle model if the CDH assumption holds.*

**Remark 2.3.** *The original IBS and IBAS schemes of Yuan et al. is described in the additive notation in bilinear groups. However, in this paper, we use the multiplicative notation instead of the additive notation for the notational simplicity.*

**Remark 2.4.** *The signature of Yuan et al.'s IBS scheme is publicly re-randomizable. If  $\sigma = (U, V, W)$  is a valid signature, then a re-randomized signature  $\sigma' = (U \cdot g^{r'}, V \cdot g_1^{r'}, W \cdot g_2^{r'})$  is also a valid one where  $r'$  is a random exponent in  $\mathbb{Z}_p^*$ .*

## 2.3 The Identity-Based Aggregate Signature Scheme

The IBAS scheme consists of **Setup**, **GenKey**, **Sign**, **Verify**, **Aggregate**, and **AggVerify** algorithms. The **Setup**, **GenKey**, **Sign**, and **Verify** algorithms of Yuan *et al.*'s IBAS scheme is the same as those of their IBS scheme. The IBAS scheme of Yuan *et al.* [17] is described as follows:

**Aggregate**( $\sigma_1, \sigma_2, S_1, S_2, PP$ ): This algorithm takes as input a signature  $\sigma_1 = (U_1, V_1, W_1)$  on a multiset  $S_1 = \{(ID_{1,1}, M_{1,1}), \dots, (ID_{1,n_1}, M_{1,n_1})\}$  of identity and message pairs, a signature  $\sigma_2 = (U_2, V_2, W_2)$  on a multiset  $S_2 = \{(ID_{2,1}, M_{2,1}), \dots, (ID_{2,n_2}, M_{2,n_2})\}$  of identity and message pairs, and the public parameters  $PP$ . It outputs an aggregate signature  $\sigma = (U = U_1 \cdot U_2, V = V_1 \cdot V_2, W = W_1 \cdot W_2)$  on the multiset  $S = S_1 \cup S_2$ .

**AggVerify**( $\sigma, S, PP$ ): This algorithm takes as input an aggregate signature  $\sigma = (U, V, W)$ , a multiset  $S = \{(ID_1, M_1), \dots, (ID_n, M_n)\}$  of identity and message pairs, and the public parameters  $PP$ . It computes  $h_i = H(ID_i \| M_i)$  for  $i = 1, \dots, n$  and checks whether  $e(V, g) \stackrel{?}{=} e(\prod_{i=1}^n H_1(ID_i)^{h_i} \cdot U, g_1)$  and  $e(W, g) \stackrel{?}{=} e(\prod_{i=1}^n H_1(ID_i) \cdot U, g_2)$ . If both equations hold, then it outputs 1. Otherwise, it outputs 0.

**Claim 2.5** ([17]). *The above IBAS scheme is existentially unforgeable under chosen message attacks in the random oracle model if the underlying IBS scheme is unforgeable under chosen message attacks.*

### 3 Forgery Attacks on the IBAS Scheme

In this section, we show that the IBS and IBAS schemes of Yuan *et al.* are not secure at all by presenting an efficient forgery algorithm. In fact, our forgery algorithm is universal since anyone who has two valid signatures on the same identity with different messages can generate a forge signature on the same identity with any message of its choice.

**Lemma 3.1.** *There exists a probabilistic polynomial-time (PPT) algorithm  $\mathcal{F}$  that can forge the IBS scheme of Yuan *et al.* except negligible probability if  $\mathcal{F}$  makes just two signature queries.*

*Proof.* The basic idea of our forgery attack is that if a forger obtains two valid signature on an identity, then a linear combination of these signatures can be another valid signature by carefully choosing scalar values. A forgery algorithm  $\mathcal{F}$  is described as follows:

1.  $\mathcal{F}$  randomly selects a target identity  $ID^*$  and two different messages  $M_1$  and  $M_2$ . It obtains a signature  $\sigma_1 = (U_1, V_1, W_1)$  on the pair  $(ID^*, M_1)$  and a signature  $\sigma_2 = (U_2, V_2, W_2)$  on the pair  $(ID^*, M_2)$  from the signature oracle.
2. It randomly selects a target message  $M^*$  for a forged signature. Next, it computes  $h_1 = H_2(ID^* \| M_1)$ ,  $h_2 = H_2(ID^* \| M_2)$ , and  $h^* = H_2(ID^* \| M^*)$ . It computes two exponents  $\delta_1, \delta_2$  that satisfy the following equation

$$\begin{bmatrix} h_1 & h_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} h^* \\ 1 \end{bmatrix} \pmod{p}.$$

Note that if  $h_1 \neq h_2$ , then  $\delta_1, \delta_2$  can be computed by using Linear Algebra since the determinant  $h_1 - h_2$  of the left matrix is not zero.

3. Finally,  $\mathcal{F}$  outputs a forged signature  $\sigma^*$  on the identity and message pair  $(ID^*, M^*)$  as

$$\sigma^* = (U^* = U_1^{\delta_1} \cdot U_2^{\delta_2}, V^* = V_1^{\delta_1} \cdot V_2^{\delta_2}, W^* = W_1^{\delta_1} \cdot W_2^{\delta_2}).$$

To finish the proof, we should show that the forger  $\mathcal{F}$  outputs a (forged) signature with non-negligible probability and the forged signature passes the verification algorithm. We known that  $\mathcal{F}$  always outputs a signature if  $h_1 \neq h_2$ . Because  $H_2$  is a collision-resistant hash function and  $M_1 \neq M_2$ , we have that  $h_1 \neq h_2$  except negligible probability. Now we should show that the forged signature is correct by the verification algorithm. Let  $r_1, r_2$  be the randomness of  $\sigma_1, \sigma_2$  respectively. The correctness of the forged signature is

easily verified as follows

$$\begin{aligned}
U^* &= U_1^{\delta_1} \cdot U_2^{\delta_2} = g^{r_1 \delta_1 + r_2 \delta_2} = g^{r^*}, \\
V^* &= V_1^{\delta_1} \cdot V_2^{\delta_2} = (H_1(ID^*)^{s_1 h_1} g_1^{r_1})^{\delta_1} \cdot (H_1(ID^*)^{s_1 h_2} g_1^{r_2})^{\delta_2} \\
&= H_1(ID^*)^{s_1(h_1 \delta_1 + h_2 \delta_2)} g_1^{r_1 \delta_1 + r_2 \delta_2} = H_1(ID^*)^{s_1 h^*} g_1^{r^*}, \\
W^* &= W_1^{\delta_1} \cdot W_2^{\delta_2} = (H_1(ID^*)^{s_2} g_2^{r_1})^{\delta_1} \cdot (H_1(ID^*)^{s_2} g_2^{r_2})^{\delta_2} \\
&= H_1(ID^*)^{s_2(\delta_1 + \delta_2)} g_2^{r_1 \delta_1 + r_2 \delta_2} = H_1(ID^*)^{s_2} g_2^{r^*}
\end{aligned}$$

where the randomness of the forged signature is defined as  $r^* = r_1 \delta_1 + r_2 \delta_2 \pmod p$ . This completes the proof.  $\square$

**Corollary 3.2.** *There exists a PPT algorithm  $\mathcal{F}$  that can forge the IBAS scheme of Yuan et al. except negligible probability if  $\mathcal{F}$  makes just two signature queries.*

The proof of this corollary is trivial from the proof of the previous Lemma since the IBAS scheme uses the IBS scheme as the underlying signature scheme. We omit the proof.

## 4 Our Analysis of the Security Proof

From the forgery attack in the previous section, it is evident that the IBS and IBAS schemes of Yuan et al. are not secure. However, Yuan et al. claimed that their IBS scheme is secure in the random oracle model under the CDH assumption by using Forking Lemma in [17]. In this section, we analyze the security proof of Yuan et al. and show that there is a critical flaw in their security proof that uses Forking Lemma.

### 4.1 The Original Proof

In this subsection, we briefly review the security proof of Yuan et al.'s IBS scheme [17] that solves the CDH problem by using Forking Lemma [2, 15].

Suppose there exists an adversary  $\mathcal{A}$  that outputs a forged signature for the IBS scheme with a non-negligible advantage. A simulator  $\mathcal{B}$  that solves the CDH problem using  $\mathcal{A}$  is given: a challenge tuple  $D = ((p, \mathbb{G}, \mathbb{G}_T, e), g, g^a, g^b)$ . Then  $\mathcal{B}$  that interacts with  $\mathcal{A}$  is described as follows:

**Setup:**  $\mathcal{B}$  chooses a random exponent  $s_2 \in \mathbb{Z}_p^*$  and maintains  $H_1$ -list and  $H_2$ -list for random oracles. It implicitly sets  $s_1 = a$  and publishes the public parameters  $PP = ((p, \mathbb{G}, \mathbb{G}_T, e), g, g_1 = g^a, g_2 = g^{s_2}, H_1, H_2)$ .

**Hash Query:** If this is an  $H_1$  hash query on an identity  $ID_i$ , then  $\mathcal{B}$  handles this query as follows: If the identity  $ID_i$  already appears in  $H_1$ -list, then it responds with the value in the list. Otherwise, it picks a random coin  $c \in \{0, 1\}$  with  $\Pr[c = 0] = \delta$  for some  $\delta$  and proceeds as follows: If  $c = 0$ , then it chooses  $t_i \in \mathbb{Z}_p^*$  and sets  $Q_i = (g^b)^{t_i}$ . If  $c = 1$ , then it chooses  $t_i \in \mathbb{Z}_p^*$  and sets  $Q_i = g^{t_i}$ . Next, it adds  $(ID_i, t_i, c, Q_i)$  to  $H_1$ -list and responds to  $\mathcal{A}$  with  $H_1(ID_i) = Q_i$ .

If this is an  $H_2$  hash query on an identity  $ID_i$  and a message  $M_i$ , then  $\mathcal{B}$  handles this query as follows: If the tuple  $(ID_i, M_i)$  already appears on  $H_2$ -list, then it responds with the value in the list. Otherwise, it randomly chooses  $h_i \in \mathbb{Z}_p^*$ , add  $(ID_i, M_i, h_i)$  to  $H_2$ -list, and responds with  $H_2(ID_i || M_i) = h_i$ .

**Private-Key Query:**  $\mathcal{B}$  handles a private key query for an identity  $ID_i$  as follows: It first retrieves  $(ID_i, t_i, c, Q_i)$  from the  $H_1$ -list. If  $c = 0$ , then it aborts the simulation since it cannot create a private key. Otherwise, it creates a private key  $SK_{ID_i} = (D_1 = g_1^{t_i}, D_2 = g_2^{t_i})$  and responds to  $\mathcal{A}$  with  $SK_{ID_i}$ .

**Signature Query:**  $\mathcal{B}$  handles a signature query on an identity  $ID_i$  and a message  $M_i$  as follows: It randomly chooses  $r' \in \mathbb{Z}_p^*$  and computes  $h = H_2(ID_i \| M_i)$ . Next, it responds to  $\mathcal{A}$  with a signature  $\sigma = (U = g^{r'} \cdot H_1(ID_i)^{-h}, V = g^{r'}, W = H_1(ID_i)^{s_2} \cdot U^{s_2})$ .

**Output:**  $\mathcal{A}$  finally outputs a forged signature  $\sigma^* = (U^*, V^*, W^*)$  on an identity  $ID^*$  and a message  $M^*$ .

To solve the CDH problem,  $\mathcal{B}$  retrieves the tuple  $(ID^*, t^*, c^*, Q^*)$  from the  $H_1$ -list. If  $c^* \neq 0$ , then it aborts since it cannot extract the CDH value. Otherwise, it obtains two valid signatures  $\sigma_1^* = (U_1^*, V_1^*, W_1^*)$  and  $\sigma_2^* = (U_2^*, V_2^*, W_2^*)$  on the same identity and message tuple  $(ID^*, M^*)$  such that  $U_1^* = U_2^*$  and  $h_1^* \neq h_2^*$  by applying Forking Lemma. That is, it replays  $\mathcal{F}$  with the same random tape but different choice of the random oracle  $H_2$ . If  $U_1^* = U_2^*$ , then we have the following equation

$$\begin{aligned} V_1^* \cdot (V_2^*)^{-1} &= H_1(ID^*)^{s_1 h_1^*} g_1^{s_1 r^*} \cdot (H_1(ID^*)^{s_1 h_2^*} g_1^{s_1 r^*})^{-1} \\ &= H_1(ID^*)^{s_1 (h_1^* - h_2^*)} = (g^{ab})^{t^* (h_1^* - h_2^*)}. \end{aligned}$$

Thus,  $\mathcal{B}$  can compute the CDH value as  $(V_1^* \cdot (V_2^*)^{-1})^{1/(t^* (h_1^* - h_2^*))}$  if  $h_1^* \neq h_2^* \pmod p$ .

## 4.2 A Non-Extractable Forgery

To extract the CDH value from forged signatures by applying Forking Lemma, it is essential for the simulator to obtain two valid signatures  $\sigma_1^*$  and  $\sigma_2^*$  such that  $U_1^* = U_2^*$  and  $h_1^* \neq h_2^*$ . By replaying a forgery with the same random tape with different choice of random oracle  $H_2$ , it is possible for a simulator to obtain two valid signatures  $\sigma_1^* = (U_1^*, V_1^*, W_1^*)$  and  $\sigma_2^* = (U_2^*, V_2^*, W_2^*)$  with  $h_1^* \neq h_2^*$  because of Forking Lemma. However, we show that the probability of  $U_1^* = U_2^*$  is negligible for some clever forgery.

**Lemma 4.1.** *If there is a PPT algorithm  $\mathcal{A}$  that can forge the IBS scheme of Yuan et al., then there is another PPT algorithm  $\mathcal{F}$  that can forge the IBS scheme with almost the same probability except that the simulator of Yuan et al. cannot extract the CDH value from the forged signatures of  $\mathcal{F}$ .*

*Proof.* The basic idea of this proof is that anyone can re-randomize the signature of Yuan et al.'s IBS scheme by using the public parameters. In this case, even though a simulator use the same random tape for Forking Lemma, a forgery output a forged signature  $\sigma^*$  on an identity  $ID^*$  and a message  $M^*$  after re-randomizing it by using the information  $h^* = H_2(ID^* \| M^*)$ . Let  $H' : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  be a collision resistant hash function that is not modeled as the random oracle. A new forgery  $\mathcal{F}$  that uses  $\mathcal{A}$  as a sub-routine is described as follows:

1.  $\mathcal{F}$  is first given  $PP$  and runs  $\mathcal{A}$  by giving  $PP$ .  $\mathcal{F}$  also handles the private key and signature queries of  $\mathcal{A}$  by using his own private key and signature oracles.
2.  $\mathcal{A}$  finally outputs a forged signature  $\sigma' = (U', V', W')$  on an identity  $ID^*$  and a message  $M^*$ .
3.  $\mathcal{F}$  computes  $h^* = H_2(ID^* \| M^*)$  and  $h' = H'(U' \| h^*)$ , and then it re-randomizes the forged signature as

$$\sigma^* = (U^* = U' \cdot g^{h'}, V^* = V' \cdot g_1^{h'}, W^* = W' \cdot g_2^{h'}).$$

4. Finally,  $\mathcal{F}$  outputs  $\sigma^* = (U^*, V^*, W^*)$  as the forged signature on an identity  $ID^*$  and  $M^*$ .

To finish the proof, we should show that the forged signature of  $\mathcal{F}$  is correct and the simulator of Yuan et al. cannot extract the CDH value from the forged signatures by using Forking Lemma. Let  $r'$  be the

randomness of  $\sigma'$ . The correctness of the forged signature is easily checked as follows

$$\begin{aligned} U^* &= U' \cdot g^{h'} = g^{r'+h'} = g^{r^*}, \\ V^* &= V' \cdot g_1^{h'} = H_1(ID^*)^{s_1 h^*} g_1^{r'+h'} = H_1(ID^*)^{s_1 h^*} g_1^{r^*}, \\ W^* &= W' \cdot g_2^{h'} = H_1(ID^*)^{s_2 h^*} g_2^{r'+h'} = H_1(ID^*)^{s_2 h^*} g_2^{r^*} \end{aligned}$$

where  $h^* = H_2(ID^* \| M^*)$  and  $r^* = r' + h'$ . To extract the CDH value from the forged signature of  $\mathcal{F}$  by using Forking Lemma, the simulator of Yuan *et al.* should obtain two valid signatures  $\sigma_1^* = (U_1^*, V_1^*, W_1^*)$  and  $\sigma_2^* = (U_2^*, V_2^*, W_2^*)$  on the same identity and message pair  $(ID^*, M^*)$  such that  $U_1^* = U_2^*$  and  $h_1^* \neq h_2^*$  after replaying  $\mathcal{F}$  with the same random tape but different choices of the hash oracle  $H_2$ . Let  $\sigma_1^* = (U_1^*, V_1^*, W_1^*)$  and  $\sigma_2^* = (U_2^*, V_2^*, W_2^*)$  be the two valid signatures obtained from  $\mathcal{F}$  by using Forking Lemma and  $\sigma'_1 = (U'_1, V'_1, W'_1)$  and  $\sigma'_2 = (U'_2, V'_2, W'_2)$  be the original signatures before the re-randomization of  $\mathcal{F}$ . If  $h_1^* \neq h_2^*$ , then  $h'_1 \neq h'_2$  except negligible probability since  $H'$  is a collision-resistance hash function and the inputs of this hash function are different. From  $h'_1 \neq h'_2$ , we have  $U_1^* \neq U_2^*$  except negligible probability since  $U'_1$  and  $U'_2$  are re-randomized with difference values  $g^{h'_1}$  and  $g^{h'_2}$  respectively. Therefore, the event that the simulator obtains two valid signatures such that  $U_1^* = U_2^*$  and  $h_1^* \neq h_2^*$  by using Forking Lemma only occurs with negligible probability. This completes our proof.  $\square$

### 4.3 Discussions

From the above analysis, we know that the original IBS scheme of Yuan *et al.* cannot be proven secure under the CDH assumption by applying Forking Lemma since the signature is publicly re-randomizable. To fix this problem, we may modify the IBS scheme to compute  $h = H_2(U \| ID \| M)$  instead of  $h = H_2(ID \| M)$  where  $U$  is the first element of a signature. In this case, the signature of the modified IBS scheme is not re-randomizable since  $U$  is given to the input of  $H_2$ . Note that our forgery attack in the previous section also does not work in this modified IBS scheme. However, this modified IBS scheme does not lead to an IBAS scheme since each  $U$  in individual signatures cannot be aggregated. Note that if each  $U$  is aggregated, then a verifier cannot check the validity of an aggregate signature since each  $U$  is not given in the aggregate signature. Therefore, there is no easy fix to solve the problem.

## 5 Conclusion

In this paper, we showed that the IBS and IBAS schemes of Yuan *et al.* are not secure at all. We first presented an efficient forgery attack on the IBS scheme and their security proof of the IBS scheme has a serious flaw. The IBAS scheme is also not secure since the security of their IBAS scheme is based on the security of their IBS scheme. Therefore, constructing an IBAS scheme with full aggregation in bilinear maps is still left as an important open problem.

## References

- [1] Jae Hyun Ahn, Matthew Green, and Susan Hohenberger. Synchronized aggregate signatures: new definitions, constructions and applications. In *ACM Conference on Computer and Communications Security*, pages 473–484, 2010.

- [2] Mihir Bellare and Gregory Neven. Multi-signatures in the plain public-key model and a general forking lemma. In Ari Juels, Rebecca N. Wright, and Sabrina De Capitani di Vimercati, editors, *CCS 2006*, pages 390–399. ACM, 2006.
- [3] Alexandra Boldyreva, Craig Gentry, Adam O’Neill, and Dae Hyun Yum. Ordered multisignatures and identity-based sequential aggregate signatures, with applications to secure routing. In Peng Ning, Sabrina De Capitani di Vimercati, and Paul F. Syverson, editors, *ACM Conference on Computer and Communications Security*, pages 276–285. ACM, 2007.
- [4] Alexandra Boldyreva, Craig Gentry, Adam O’Neill, and Dae Hyun Yum. Ordered multisignatures and identity-based sequential aggregate signatures, with applications to secure routing. Cryptology ePrint Archive, Report 2007/438, 2010. <http://eprint.iacr.org/2007/438>.
- [5] Dan Boneh, Craig Gentry, Ben Lynn, and Hovav Shacham. Aggregate and verifiably encrypted signatures from bilinear maps. In Eli Biham, editor, *EUROCRYPT 2003*, volume 2656 of *Lecture Notes in Computer Science*, pages 416–432. Springer, 2003.
- [6] Sanjam Garg, Craig Gentry, and Shai Halevi. Candidate multilinear maps from ideal lattices. In Thomas Johansson and Phong Q. Nguyen, editors, *EUROCRYPT 2013*, volume 7881 of *Lecture Notes in Computer Science*, pages 1–17. Springer, 2013.
- [7] Craig Gentry and Zulfikar Ramzan. Identity-based aggregate signatures. In Moti Yung, Yevgeniy Dodis, Aggelos Kiayias, and Tal Malkin, editors, *PKC 2006*, volume 3958 of *Lecture Notes in Computer Science*, pages 257–273. Springer, 2006.
- [8] Michael Gerbush, Allison B. Lewko, Adam O’Neill, and Brent Waters. Dual form signatures: An approach for proving security from static assumptions. In Xiaoyun Wang and Kazue Sako, editors, *ASIACRYPT 2012*, volume 7658 of *Lecture Notes in Computer Science*, pages 25–42. Springer, 2012.
- [9] Susan Hohenberger, Amit Sahai, and Brent Waters. Full domain hash from (leveled) multilinear maps and identity-based aggregate signatures. In Ran Canetti and Juan A. Garay, editors, *CRYPTO 2013*, volume 8042 of *Lecture Notes in Computer Science*, pages 494–512. Springer, 2013.
- [10] Jung Yeon Hwang, Dong Hoon Lee, and Moti Yung. Universal forgery of the identity-based sequential aggregate signature scheme. In Wanqing Li, Willy Susilo, Udaya Kiran Tupakula, Reihaneh Safavi-Naini, and Vijay Varadharajan, editors, *ASIACCS 2009*, pages 157–160. ACM, 2009.
- [11] Kwangsu Lee, Dong Hoon Lee, and Moti Yung. Aggregating cl-signatures revisited: Extended functionality and better efficiency. In Ahmad-Reza Sadeghi, editor, *FC 2013*, volume 7859 of *Lecture Notes in Computer Science*, pages 171–188. Springer, 2013.
- [12] Kwangsu Lee, Dong Hoon Lee, and Moti Yung. Sequential aggregate signatures with short public keys: Design, analysis and implementation studies. In Kaoru Kurosawa and Goichiro Hanaoka, editors, *PKC 2013*, volume 7778 of *Lecture Notes in Computer Science*, pages 423–442. Springer, 2013.
- [13] Steve Lu, Rafail Ostrovsky, Amit Sahai, Hovav Shacham, and Brent Waters. Sequential aggregate signatures and multisignatures without random oracles. In Serge Vaudenay, editor, *EUROCRYPT 2006*, volume 4004 of *Lecture Notes in Computer Science*, pages 465–485. Springer, 2006.



- [14] Anna Lysyanskaya, Silvio Micali, Leonid Reyzin, and Hovav Shacham. Sequential aggregate signatures from trapdoor permutations. In Christian Cachin and Jan Camenisch, editors, *EUROCRYPT 2004*, volume 3027 of *Lecture Notes in Computer Science*, pages 74–90. Springer, 2004.
- [15] David Pointcheval and Jacques Stern. Security arguments for digital signatures and blind signatures. *J. Cryptology*, 13(3):361–396, 2000.
- [16] Dominique Schröder. How to aggregate the cl signature scheme. In Vijay Atluri and Claudia Díaz, editors, *ESORICS 2011*, volume 6879 of *Lecture Notes in Computer Science*, pages 298–314. Springer, 2011.
- [17] Yumin Yuan, Qian Zhan, and Hua Huang. Efficient unrestricted identity-based aggregate signature scheme. *PLoS ONE*, 9(10):e110100, 2014.